

ROTOR INTERNAL FRICTION INSTABILITY

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Two aspects of internal friction affecting stability of rotating machines are discussed in this paper. The first role of internal friction consists of decreasing the level of effective damping during rotor subsynchronous and backward precessional vibrations caused by some other instability mechanisms. The second role of internal friction consists of creating rotor instability, i.e., causing self-excited subsynchronous vibrations. Experimental test results document both of these aspects.

1. INTRODUCTION

In rotating machines, damping effects are conventionally split into two categories: external and internal damping. The term "external" refers to the stationary elements and rotor environment, as they are "external" to the rotor. External damping is related to energy dissipation due to friction occurring between stationary and rotating elements, and/or fluid dynamic resistance in the rotor environment (mostly in fluid-lubricated bearings). External damping is also supplied by material damping of supports. External damping forces depend on the rotor absolute velocity of vibration, and their effect on the rotor vibration is usually welcome -- they provide stabilizing factors.

The term "internal" refers to the rotating elements, including the rotor itself. The same physical phenomena characterize internal and external damping. Internal damping forces are due to material damping in rotating and vibrating elements (mostly shafts) and friction between rotating parts (such as joint couplings and shrink-fitted disks on shafts). As internal damping occurs in the elements involved in rotating motion, the internal damping forces depend on relative velocity, i.e., on the difference between the absolute vibration velocity and the rotative speed. Thus the relative velocity may be "positive" (following absolute velocity when rotative speed is low) or "negative" (opposing absolute velocity when rotative speed is high). The corresponding internal damping forces can, therefore, act as a stabilizing (adding to the external damping to increase the total effective damping in the system) or a destabilizing factor (subtracting from the external damping to decrease the total effective damping of the system).

The term "damping" is traditionally related to the stabilizing effects created by the irreversible conversion of kinetic energy to heat. Being physically related to the same mechanism, rotor internal damping plays an additional role in rotating machines -- it opposes the external damping and actually transfers the rotational energy into vibrations. That is why the term "damping" does not fit well in this situation; by contrast, the term "internal friction" does not introduce ambiguity concerning stabilization.

Internal friction has been recognized as a cause of unstable rotor motion for more than 50 years [1-7]. Since the first description of internal friction-related in-

stability, many other rotor destabilizing factors have been identified, such as rub or fluid dynamic effects in fluid-flow machines and/or bearings and seals. The latter effects are much stronger than the internal friction effects and very often observed in the performance of rotating machines. They result in subsynchronous vibrations (rotative speed higher than vibrational frequency). Internal friction is now very seldom identified as a main cause of rotor unstable motion. However, internal friction plays a negative role by reducing the system-effective damping for forward subsynchronous and backward vibrations caused by other destabilizing factors.

In this paper two aspects of rotor internal friction are discussed: the first, a damping-reducing effect and the second, a cause of instability and self-excited vibrations. The experimental test results document both aspects.

2. ROTOR MODEL WITH INTERNAL FRICTION

To simplify the considerations, a symmetric rotor will be discussed. Dynamic behavior of a rotor in its lateral mode of vibrations is usually represented by a set of linear ordinary differential equations. For each mode, the set of equations reduces to two, which for a symmetric rotor can be presented in the form of one complex variable equation [8]:

$$M\ddot{z} + D\dot{z} + Kz + \frac{F(\dot{z} - j\omega z)}{|\Omega - \omega|} = 0, \quad z(t) = x(t) + jy(t), \quad \Omega \neq \omega, \quad j = \sqrt{-1} \quad (1)$$

where x, y are rotor horizontal and vertical deflections correspondingly, they describe the rotor precessional motion. M is the rotor mass; D is the external viscous damping coefficient; K is the rotor stiffness, including the shaft and pedestal stiffness. The rotor parameters M, D, K are generalized parameters corresponding to each separate mode; ω is rotative speed. The equation (1) may, in particular, describe rotor at its first mode. Eq.(1) contains frequency Ω of the resulting precessional motion, unknown a priori; $|\Omega - \omega|$ is the value of the shaft actual bending frequency. It has been introduced to the rotor model (1) following the way by which hysteretic damping is usually included in models of mechanical systems: a viscous damping coefficient is replaced by a product $K\eta/\Omega^*$, where K is stiffness coefficient, η is loss factor, and Ω^* represents the frequency of elastic element deformation. In the case of rotating shaft, the frequency of deformation is equal to a difference between rotative speed and frequency of precession. Note that for forward low frequency precessions, the frequency of shaft deformation is lower than rotative frequency. For backward precessions, the frequency of shaft deformation is a sum of rotative speed and frequency of precession. For circular synchronous precession ($\Omega = \omega$), the shaft is "frozen" into a fixed bow shape, so that internal friction does not act.

In Eq.(1) F (assumed positive) is internal friction function. For shaft material hysteretic damping F is constant and equal to $K\eta$. For the synchronous precession $F=0$. Generally, however, F can be a nonlinear function of z and \dot{z} [4, 6, 9].

For $F = \text{const}$ and Ω supposed constant, substituting $z = z_r e^{st}$, the eigenvalue problem of (1) yields four eigenvalues (satisfying the degrees of freedom of eq. (11))*:

$$s = -\delta \pm (1/\sqrt{2})[\sqrt{\delta^2 - K/M + \sqrt{\Delta}} \pm j\sqrt{K/M - \delta^2 + \sqrt{\Delta}}] \quad (2)$$

where

$$\delta = (D + F/|\Omega - \omega|)/(2M), \quad \Delta = (\delta^2 - K/M)^2 + F^2\omega^2/[M^2(\Omega - \omega)^2]$$

The real parts of (2) are non-positive, i.e., the system (1) is stable when

$$F^2\omega^2 M/K \leq (D|\Omega - \omega| + F)^2 \quad (3)$$

which for $\omega > 0$ yields the following conditions:

For $\omega^2 < K/M$ the rotor pure rotational motion is stable.

For $\omega^2 > K/M$ it is stable only if $|F| \leq D|\Omega - \omega|/[\omega/\sqrt{K/M} - 1]$ (4)

At a threshold of stability, i.e., when $|F| = D|\Omega - \omega|/[\omega/\sqrt{K/M} - 1]$ the eigenvalues reduce to

$$s = \pm j\sqrt{K/M} \quad (5)$$

The rotor motion is purely periodic with the natural frequency determined by stiffness and mass (for the stable motion below the threshold of stability, the frequency is slightly lower than $\sqrt{K/M}$, due to damping).

If the stability condition is not satisfied and F exceeds the limits (4), then rotor pure rotational motion is unstable. The linear model (1) is not adequate anymore, as for high lateral deflections nonlinear factors become significant. Nonlinear factors eventually lead to a limit cycle of self-excited vibrations. The latter usually occur with the lowest natural frequency determined by the linear model, as the nonlinearities have very minor influence on frequency. With high amount of probability, the frequency Ω can be, therefore, equal to the rotor first natural frequency:

$$\Omega = \pm \sqrt{K_1/M_1} \quad (6)$$

*Solving the quadratic in s gives

$$s = -\left(\frac{D + \phi}{2M}\right) \pm \sqrt{\left(\frac{D + \phi}{2M}\right)^2 - \frac{K}{M} F j \frac{\phi \omega}{M}}$$

Expanding $\sqrt{a + jb} = c + jd$ and solving for (c, d) gives

$$\pm \sqrt{a \pm jb} = \pm \frac{1}{\sqrt{2}} \left[\sqrt{a + \sqrt{a^2 + b^2}} \pm j \sqrt{-a + \sqrt{a^2 + b^2}} \right]$$

Substituting

$$a \equiv \left(\frac{D + \phi}{2M}\right)^2 - \frac{K}{M} \quad \text{and} \quad b \equiv \frac{\phi \omega}{M}$$

gives four roots:

$$s = -\left(\frac{D + \phi}{2M}\right) \pm \frac{1}{\sqrt{2}} \left[\sqrt{a + \sqrt{a^2 + b^2}} \pm j \sqrt{-a + \sqrt{a^2 + b^2}} \right]$$

where the index "1" refers to the first bending mode. If the model (1) describes the first mode, the stability condition (4) reduces to

$$|F| < D\sqrt{K_1/M_1} \equiv 2K_1\xi_1 \quad (7)$$

where ξ_1 is the damping factor of the first mode.

For the hysteretic damping, $F = K_1\eta_1$ and the inequality (7) yields

$$\eta_1 \leq 2\xi_1 \quad (8)$$

i.e., for stability the shaft loss factor has to be lower than the double of external damping factor.

The modal approach to the rotor modelization permits evaluation of the stability conditions for several modes. For example, the inequality (4) for the i -th mode, (index " i ") is:

$$|F| \leq D_i |\sqrt{K_1/M_1} - \omega| / [\omega / \sqrt{K_i/M_i} - 1] \quad (9)$$

Figure 1 illustrates the condition in which the same amount of internal friction may cause the first mode to be stable and the third mode unstable. This condition takes place when the modal damping ratio is sufficiently high, $D_1/D_3 > \sqrt{K_3/M_3}/\sqrt{K_1/M_1}$ and when the rotative speed exceeds a specific value, i.e.:

$$\omega > \frac{D_1/D_3 - 1}{D_1\sqrt{M_3/K_3}/D_3 - \sqrt{M_1/K_1}} \quad (10)$$

3. ROTOR EFFECTIVE DAMPING REDUCTION DUE TO INTERNAL FRICTION

Assume that the rotor performs steady nonsynchronous precessional, self-excited vibration with frequency Ω . This vibration occurs due to any instability mechanism (for instance, it may be oil whip). It means that the rotor motion can be presented in the form

$$z = Ae^{j\Omega t} \quad (11)$$

where A is an amplitude of the self-excited vibrations. Introducing (11) into (1) gives

$$-M\Omega^2 + Dj\Omega + K + F(\Omega - \omega)/|\Omega - \omega| = 0$$

The real part of this expression yields the frequency. The imaginary part relates to the system damping. The external damping term, $D\Omega$, is now completed by the term expressing internal friction:

$$D\Omega \rightarrow D\Omega + F(\Omega - \omega)/|\Omega - \omega|$$

or

$$D \rightarrow \begin{cases} D + F/\Omega & \text{for } \omega < \Omega \text{ (supersynchronous precession)} \\ D - F/\Omega & \text{for } \omega > \Omega \text{ (subsynchronous and backward precession)} \end{cases} \quad (12)$$

For supersynchronous precession internal friction adds to external damping and increases its level. For subsynchronous and backward precession, the internal friction reduces the level of "positive" stabilizing damping in the rotor system by the amount F/Ω . Taking into account that $\Omega = \sqrt{K/M}$, for subsynchronous precession the rotor effective damping factor decreases by the following amount:

$$\xi \rightarrow \xi - F/(2K) \quad (13)$$

It also means that the Amplification Factor Q increases:

$$Q \rightarrow Q/(1-FQ/K) \quad (14)$$

If, for instance, the Synchronous Amplification Factor is 5 and internal friction is due to shaft material hysteretic damping with loss factor $\eta = 0.06$ ($F=K\eta$), then the Subsynchronous Amplification Factor increases to 7.14 (the Supersynchronous Amplification Factor decreases to 3.85).

Note that the decrease of the "positive" external damping for rotor subsynchronous vibrations does not depend on the form of the function F (constant or displacement dependent).

In practical observations of rotating machine dynamic behavior, it has very often been noticed that subsynchronous vibrations are characterized by much higher amplitudes than any super-synchronous vibrations. There are many different causes of subharmonic vibrations in rotating machines. In each case, however, the role of internal friction opposing and decreasing the level of external, stabilizing damping is very important. Although not a primary cause of instability, internal friction often promotes subsynchronous vibrations and causes an increase of amplitudes. Figures 2, 3, and 4 illustrate dynamic responses of some unstable rotating machines. The self-excited, subsynchronous vibration amplitudes are much higher than amplitudes of synchronous and supersynchronous vibrations. More examples are given in [8].

The rotor model considered in this paper is symmetric; therefore, the synchronous precession is expected to be circular. In the case of circular synchronous precession at constant rotative speed, the bent shaft precesses "frozen" and is not a subject of periodic deformation. The internal friction does not act. The regular circular synchronous precession of real rotors very seldom occurs, however; usually nonsymmetry in the rotor and/or supporting system results in the elliptical synchronous precession. In this case, the bent shaft is not "frozen," but deforms with the frequency two times higher than the rotative speed. The internal friction then brings a "positive" effect: it adds to the external damping.

4. SELF-EXCITED VIBRATIONS DUE TO INTERNAL FRICTION

If in the equation (1), F is given in the form of a nonlinear function of the rotor radial displacement $|z|$, velocity of the radial displacement $d|z|/dt$ and relative angular velocity $\omega - \dot{\theta}$, [4,6] where

$$\theta(t) = \arctan [y(t)/x(t)], \quad |z| = \sqrt{x^2 + y^2} \quad (15)$$

i.e., $F = F(|z|, d|z|/dt, \omega - \dot{\theta})$ then the rotor model (1) allows for the following particular solution

$$z(t) = B e^{j\Omega t} \quad (16)$$

where B and Ω are constant amplitude and frequency of the circular precessional self-excited vibrations correspondingly. They can be found from the following algebraic relation yielded by (1) and (16):

$$-M\Omega^2 + D j\Omega + K + jF(B, 0, \omega - \Omega)(\Omega - \omega)/|\Omega - \omega| = 0 \quad (17)$$

The nonlinear differential function F becomes nonlinear algebraic function.

Bolotin [9] quotes several forms of internal friction function F ; for instance, for a shrink-fitted disk on the shaft, the internal friction nonlinear function has the following form:

$$F(|z|, d|z|/dt, \omega - \dot{\theta}) = \frac{C_1 |z|^n}{C_2 + (\omega - \dot{\theta})^m} \quad (18)$$

where $C_1 > 0$, $C_2 > 0$, n , m are specific constant numbers. In case of the function (18) equation (17) for the first mode yields

$$\Omega = \sqrt{K_1/M_1}, \quad B = \{[C_2 + (\omega - \sqrt{K_1/M_1})^m] D\sqrt{K_1/M_1} / C_1\}^{1/n} \quad (19)$$

Since C_1 and C_2 are positive, the solution (16) with amplitude (19) exists for $\omega > \sqrt{K_1/M_1}$ only. This means that the self-excited vibrations (16) exist for sufficiently high rotative speed.

5. INTERNAL FRICTION EXPERIMENT

During balancing of the three-disk rotor rig (Fig. 5), an appearance of self-excited vibrations at the rotative speed above third balance resonance have been noticed (Fig. 6). The frequency of these self-excited vibrations exactly equals first natural frequency. The self-excited vibrations disappear for higher rotative speed. It was noted that when balancing weights, which affect the balance state for the third mode, were removed, causing a significant increase of the amplitude of the synchronous vibration at the third mode, the self-excited vibrations nearly disappeared (the amplitude decreased from 1.8 to 0.4 mils p/p, compare Figs. 6 and 8). It appeared that the energy from self-excited vibrations was transferred to the synchronous vibrations. Higher rotor deflection due to unbalance evidently caused some substantial modifications in the self-excitation mechanism.

Since there was no other obvious reason for the self-excitation, internal friction (of the shaft material and disk/shaft joints) was blamed for the appearance of these self-excited vibrations. To prove this supposition, an increase of the rotor internal friction was attempted. Half of the shaft was covered with a 4-mil-thick layer of damping material, commonly used for vibration control (acrylic adhesive ISD-112, 3M Company). Applied to the rotating shaft, the damping material increases the internal friction and magnifies the self-excitation effect. The expected result was confirmed: the amplitude of the self-excited vibrations increased from 0.4 to 0.7 mils p/p (compare Figs. 8 and 9).

The self-excited vibrations disappeared completely when the disks were eventually welded to the shaft, and the damping tape was removed.

The question of why the self-excited vibrations occur at the rotative speed ~7150 rpm and disappear in the higher range of speeds has not been answered. Nor was the internal damping function identified. The analysis presented in Section 2 gives, however, some indications that a nonlinear internal damping function may cause rotor instability in a limited range of rotative speeds. Figure 10 presents the stability chart for three modes.

6. CONCLUDING REMARKS

This paper discusses two important aspects of internal friction in rotating ma-

chines. Firstly, the internal friction in rotating elements causes a decrease of the amount of effective damping in the rotor system. This effective damping reduction occurs during rotor subsynchronous and backward precessional motion, which may be caused by any instability/self-excitation mechanism (such as rub or fluid flow dynamic forces). While usually not a primary cause of instability, internal friction promotes unstable motion and affects the value of self-excited vibration amplitudes.

Secondly, the internal friction occasionally is a major cause of rotor self-excited vibrations due to incorrect shrink fits, loosening of shrink fits by differential thermal growth, or by mechanical fatigue. Known for more than 50 years as a contributor to rotor instability, the internal friction analytical model has not, however, yet been adequately identified.

This paper documents experimentally these two aspects of internal friction in rotating machines and gives a qualitative description of the dynamic phenomena associated with rotor internal friction.

SYMBOLS

| | | | |
|-----------------|--|----------|---|
| A,B | -- Amplitudes of self-excited vibrations | s | -- Eigenvalue |
| D | -- External damping coefficient | $z=x+jy$ | -- Rotor radial deflection (x-horizontal, y-vertical) |
| F | -- Internal friction function | ζ | -- External damping factor |
| $j = \sqrt{-1}$ | | η | -- Loss factor |
| K,M | -- Rotor generalized (modal) stiffness and mass coefficients | θ | -- Angle of precessional motion |
| Q | -- Amplification Factor | ω | -- Rotative speed |
| | | Ω | -- Angular speed of precession |

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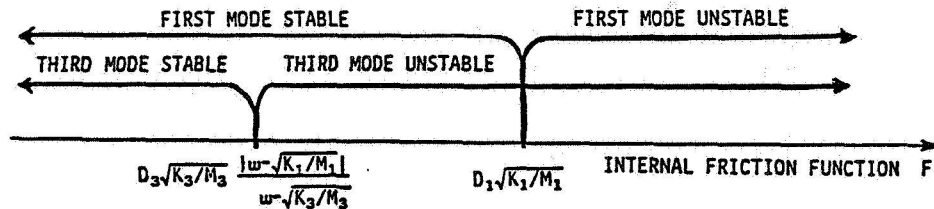


Figure 1. - Regions of stability for rotor first and third modes.

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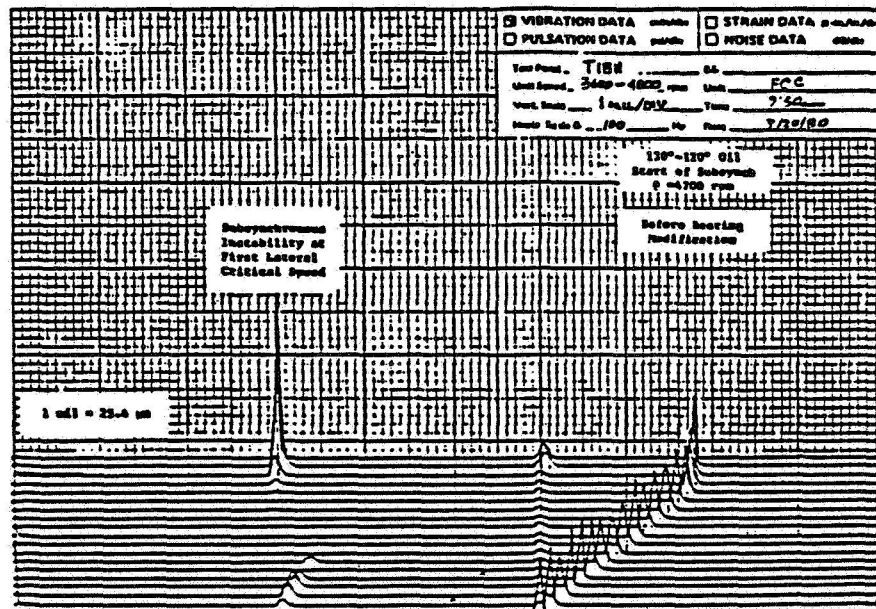


Figure 2. - Cascade spectrum of steam turbine vibrational response, indicating high subsynchronous vibrations. Data courtesy of J.C. Wachel [10].

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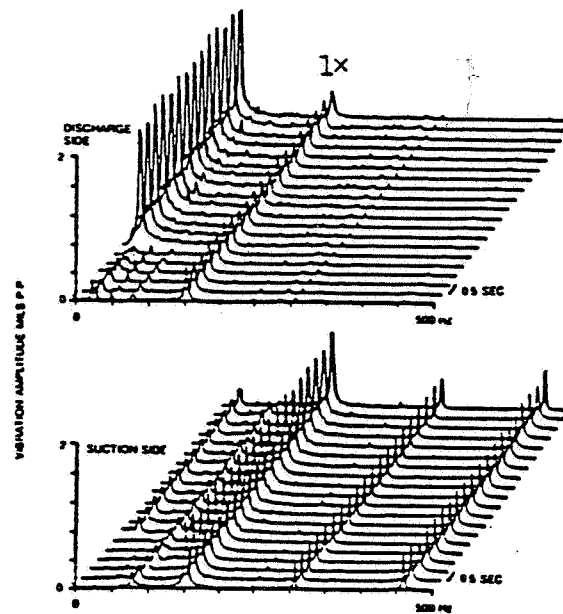


Figure 3. - Time histories of six-stage compressor at 9220 rpm. Subsynchronous vibrations due to destabilizing dynamic forces generated on last stage. Data courtesy of P.L. Ferrara [11].

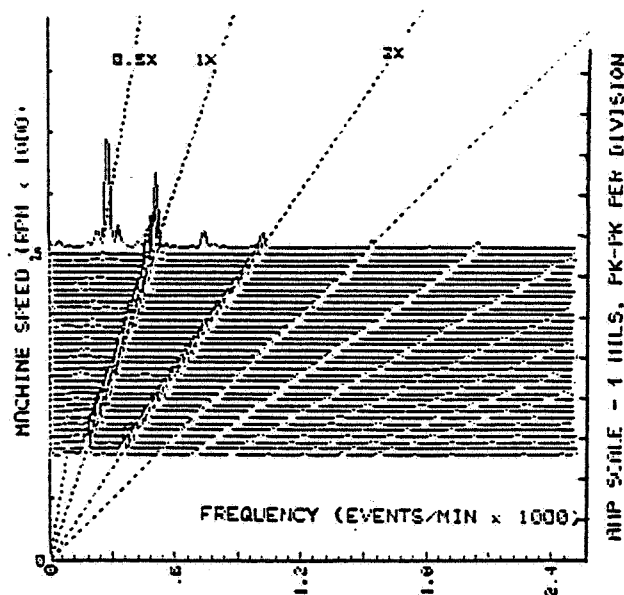


Figure 4. - Cascade plot of electric motor response during shutdown. At running speed of 510 rpm (below first balance resonance), high half-speed vibrations present due to electromagnetic field unbalance [12].

1. BASE PLATE
2. MOTOR
3. FLEXIBLE COUPLING
4. RIGID BEARING
5. KEYPHASOR DISK
6. X-Y NONCONTACTING PROBES
7. STEEL DISKS
8. STEEL SHAFT

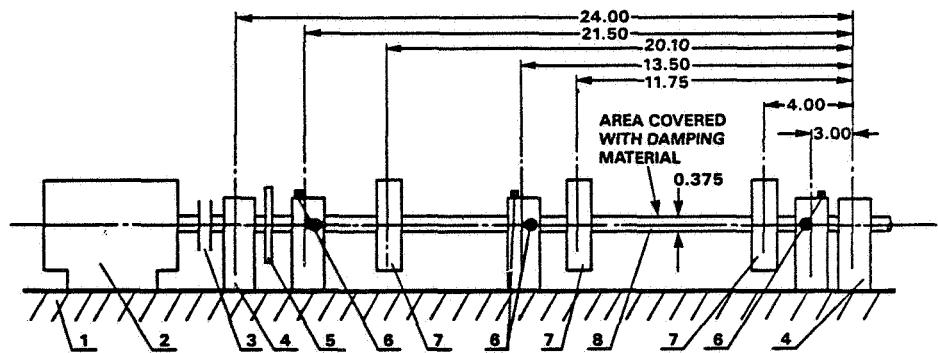


Figure 5. - Three-disk rotor rigidly supported. Disks attached to rotor by radial screws.
(Dimensions are in inches.)

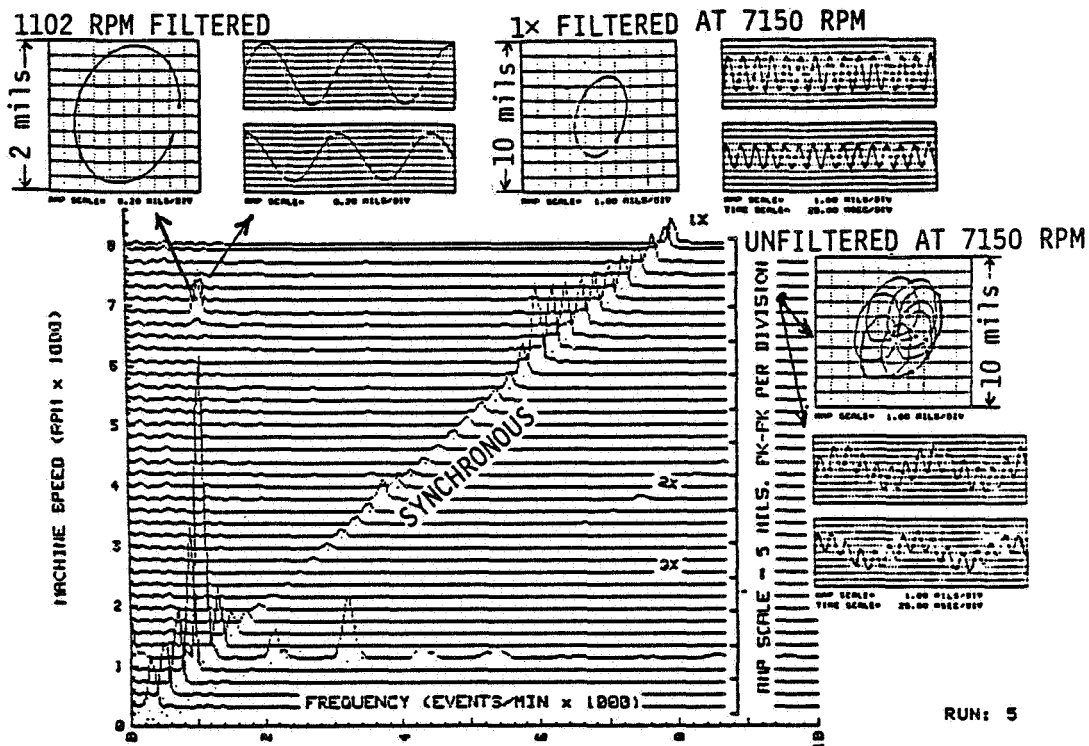


Figure 6. - Startup response of three-disk rotor measured at midspan position. Cascade spectrum and orbits indicate existence of subsynchronous self-excited vibrations.

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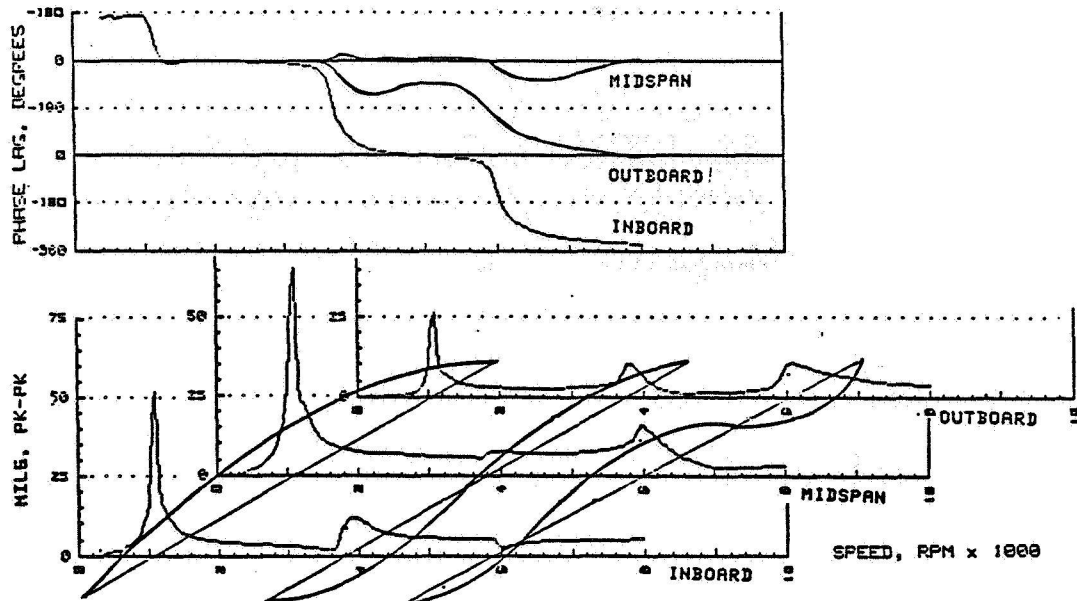


Figure 7. - Bode plots of 1x (synchronous) filtered rotor response and rotor mode shapes. Original state of unbalance.

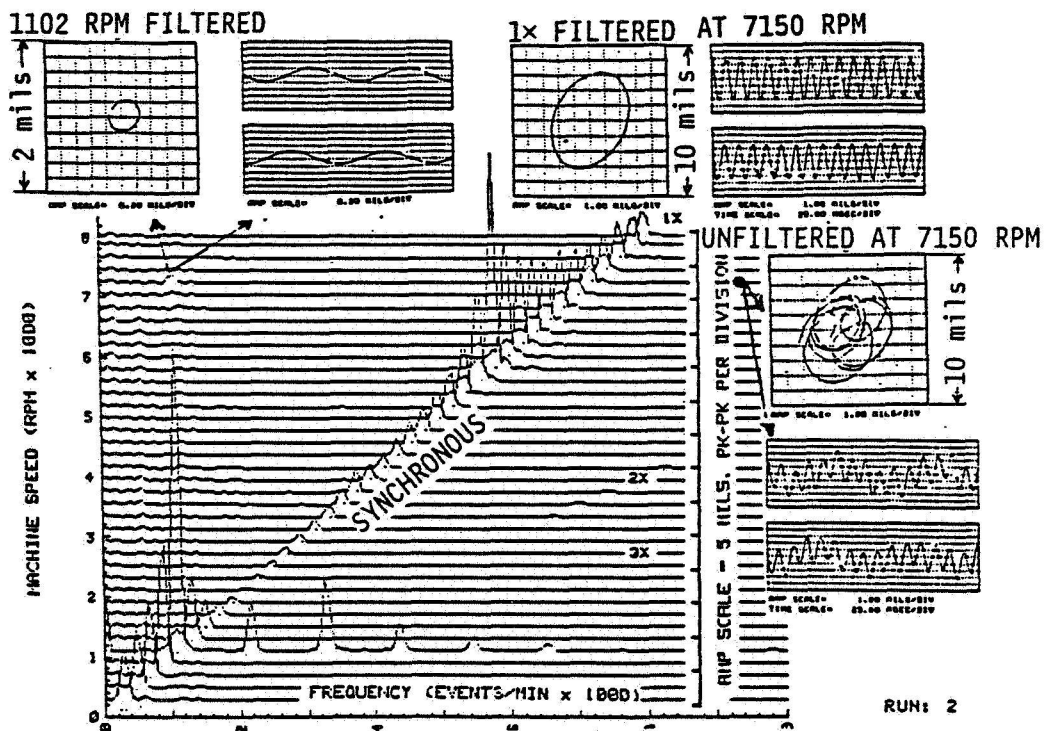


Figure 8. - Cascade spectrum and orbits of unbalanced shaft: Decrease of subsynchronous self-excited vibrations. Data from midspan position.

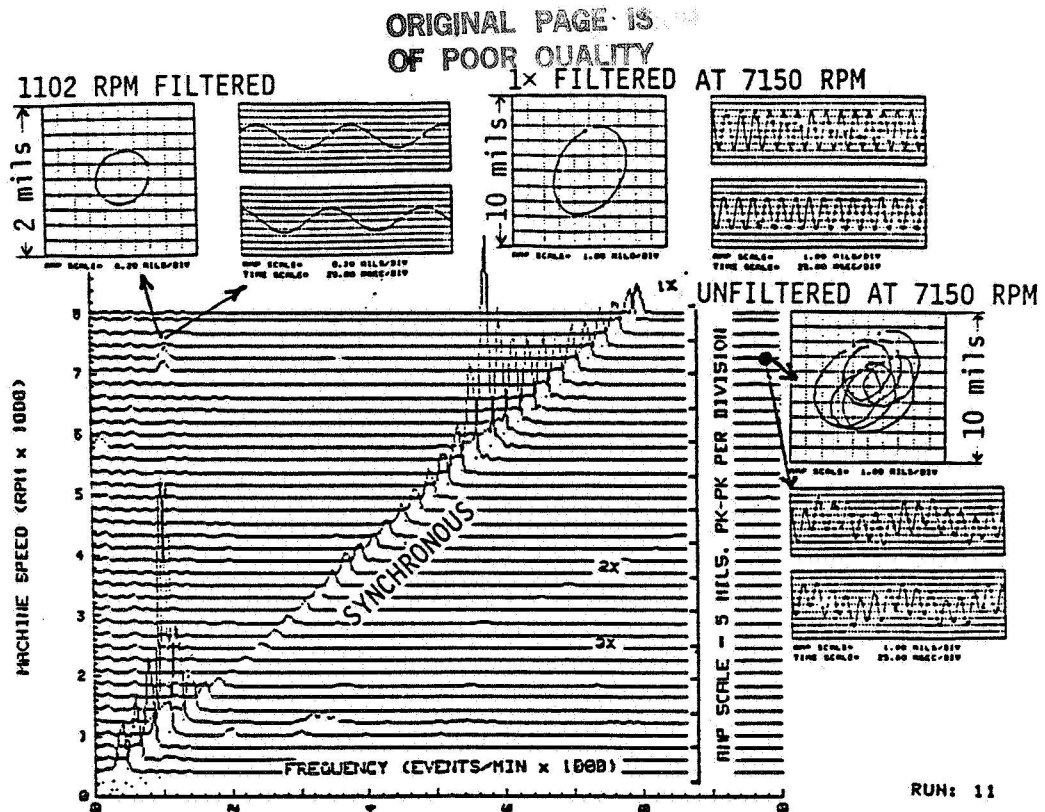


Figure 9. - Cascade spectrum and orbits of unbalanced rotor measured at midspan position. Shaft covered with damping material. Increase of subsynchronous self-excited vibrations.

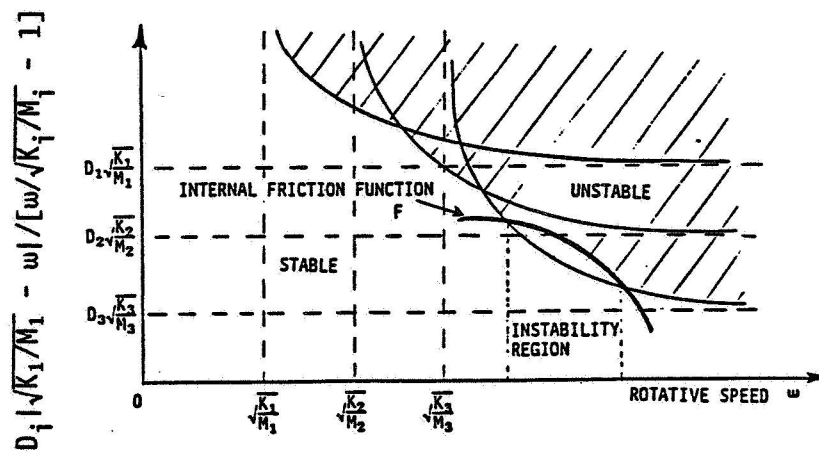


Figure 10. - Rotor stability chart for three modes (inequality (9)).